

The Role of Under-determined Approximations  
In Engineering and Science Application

from

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**ABSTRACT**

There is currently a great deal of interest in using response surfaces in the optimization of aircraft performance. The objective function and/or constraint equations involved in these optimization problems may come from numerous disciplines such as structures, aerodynamics, environmental engineering, etc. In each of these disciplines, the mathematical complexity of the governing equations usually dictates that numerical results be obtained from large computer programs such as a finite element method program. Thus, when performing optimization studies, response surfaces are a convenient way of transferring information from the various disciplines to the optimization algorithm as opposed to bringing all the sundry computer programs together in a massive computer code.

Response surfaces offer another advantage in the optimization of aircraft structures. A characteristic of these types of optimization problems is that evaluation of the objective function and response equations (referred to as a functional evaluation) can be very expensive in a computational sense. Because a great number of functional evaluations may be required in the solution of a typical engineering optimization problem, optimization of aircraft performance can require a large computing effort. Response surfaces may provide increased computational efficiency for these types of engineering optimization problems. Instead of performing exact functional evaluations during the optimization process, approximations to response can be initially developed and the approximations then optimized. Development of the approximations requires a number of initial functional evaluations. Here, however, parallel processing can provide increased computational efficiency which may speed up the total computational process.

Because of the computational expense in obtaining functional evaluations, the present study was undertaken to investigate under-determined approximations. An under-determined approximation is one in which there are fewer training pairs (pieces of information about a function) than there are undetermined parameters (coefficients or weights) associated with the approximation. Both polynomial approximations and neural net approximations were examined. Three main example problems were investigated.

In Example 1, a function of one design variable was considered. Various order polynomial approximations and neural net approximations were developed. Typical curves showing approximations of the function are included at the end of this abstract. The significant finding from this example is that under-determined approximations yield non-unique approximations. The approximation obtained depends upon such factors as initial assumed values of the undetermined parameters associated with the approximation and on the training algorithm employed.

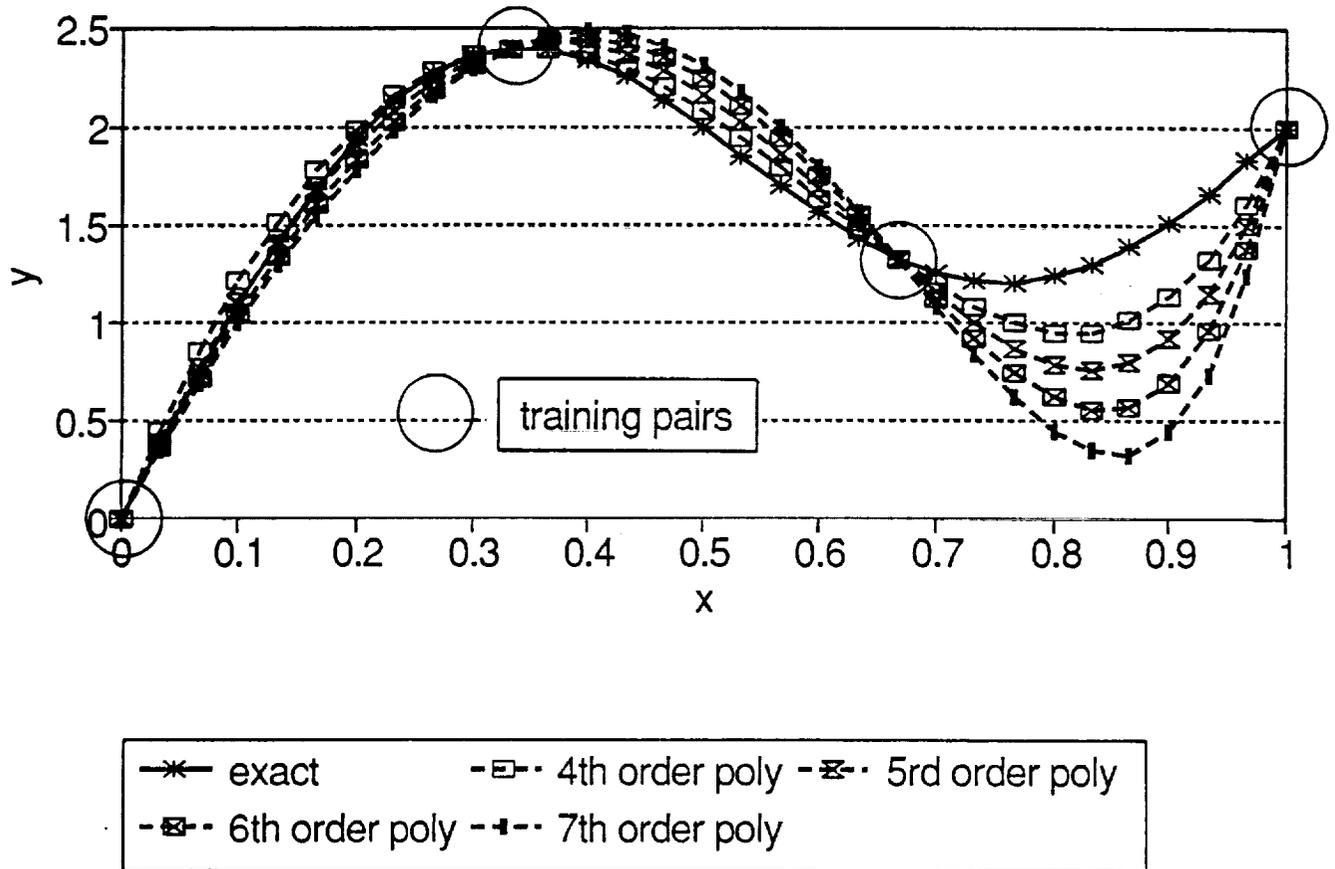
In Example 2, a function of two design variables was considered. A contour plot of the function (the banana function) is given at the end of this abstract. Here again, under-determined approximations gave non unique approximations. A figure is presented at the end of this abstract showing the variations in a parameter  $v_c$ , a measure of how well the approximation fits the function over a region of interest, for three trainings of a neural net approximation. One can see that for under-determined approximations, there is a large variability in the results which can be obtained

In Example 3, a 35 bar truss with 4 design variables was considered. Under-determined neural net approximations were considered. A figure at the end on this abstract shows that these under-determined approximations have more variability than exactly-determined or over-determined approximations.

The findings of this study are very important to work going on at NASA and in the aerospace industry. A number of recent papers have appeared reporting that under-determined approximations were being developed and used in optimization studies. This study points out that the use of under-determined approximations should be discouraged because the approximations thus obtained, while they may satisfy the training pairs, are not unique and may yield very poor approximations over a region of interest.

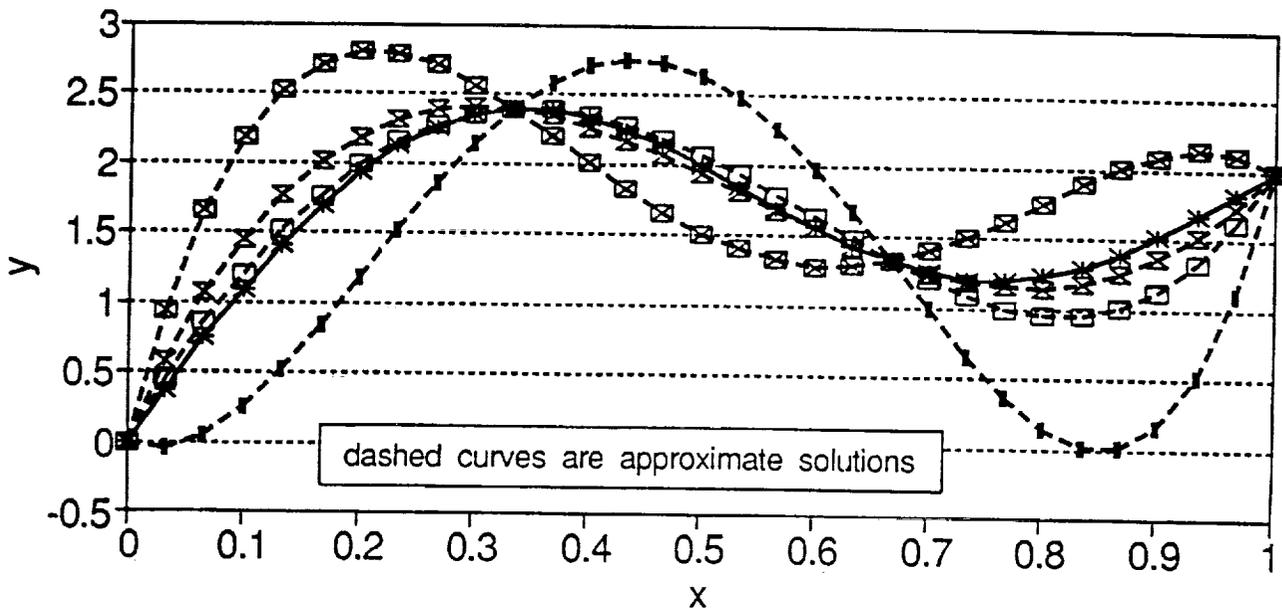
# One Dimensional Example

various polynomials, 4 training pairs



# One Dimensional Example

4th order polynomial, 4 training pairs



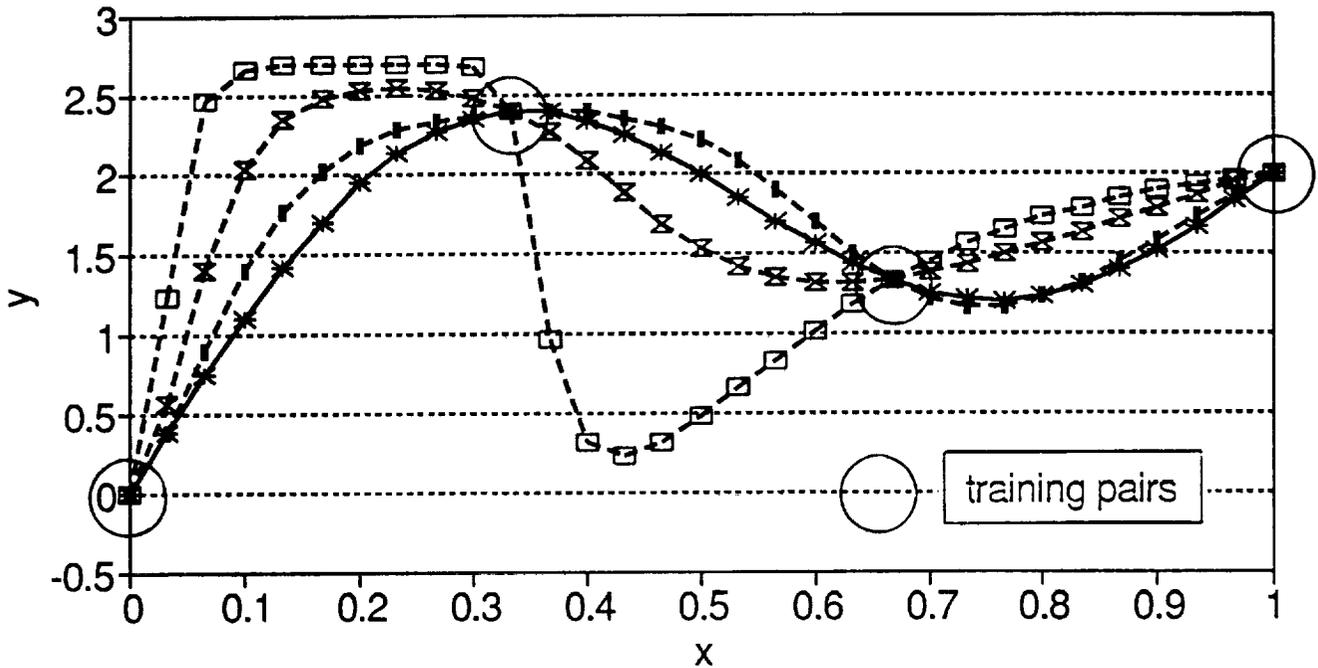
As approximation is under-determined, approximate solution is not unique.

\*— exact    -□- w=0    -x- w=.25  
 -◇- w=1.    -+- w=-1.

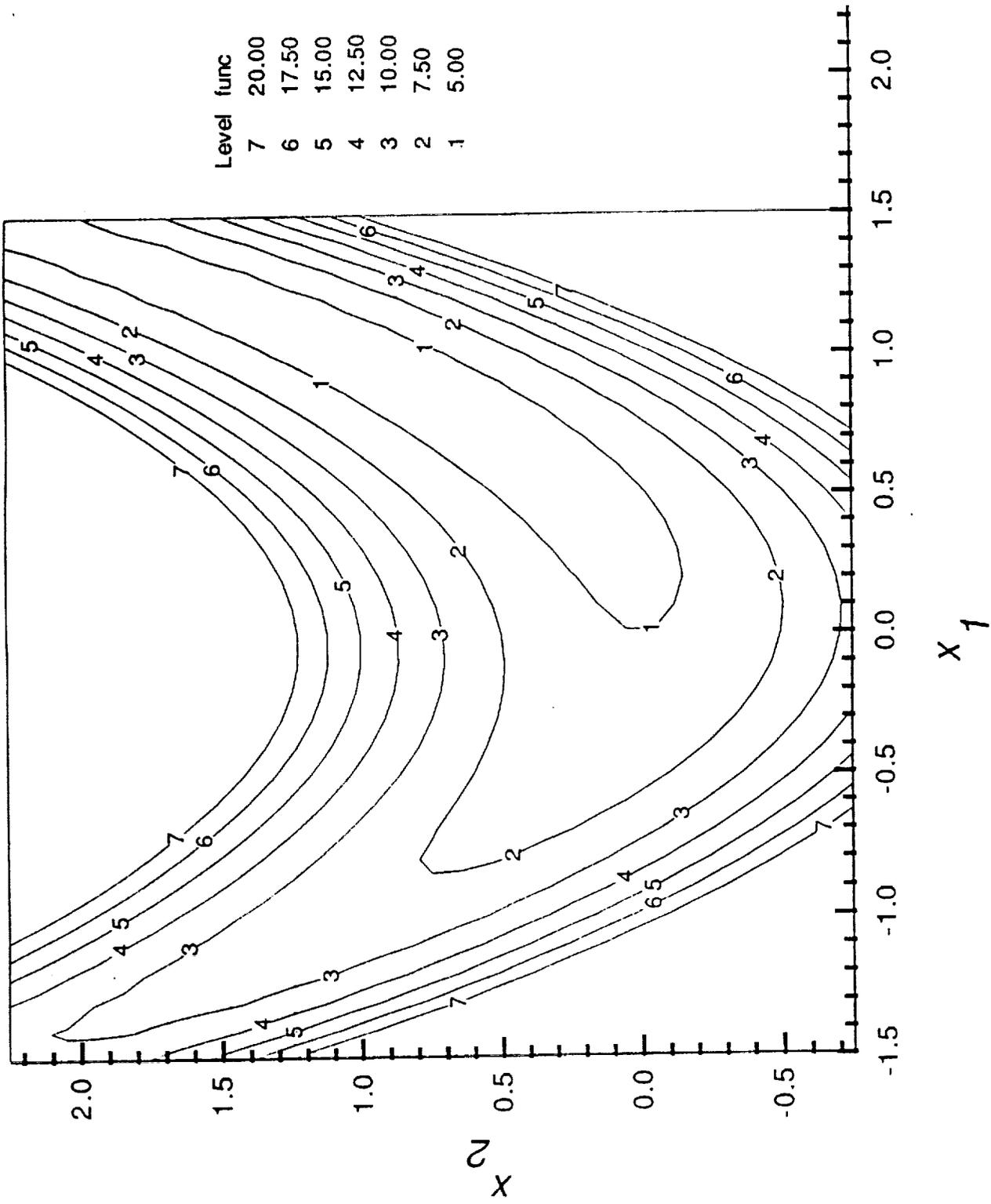
5. One Dimensional Example, various solutions using a 4th order polynomial

# One Dimensional Example

Neural Nets,  $ih=6$ , 4 training pairs



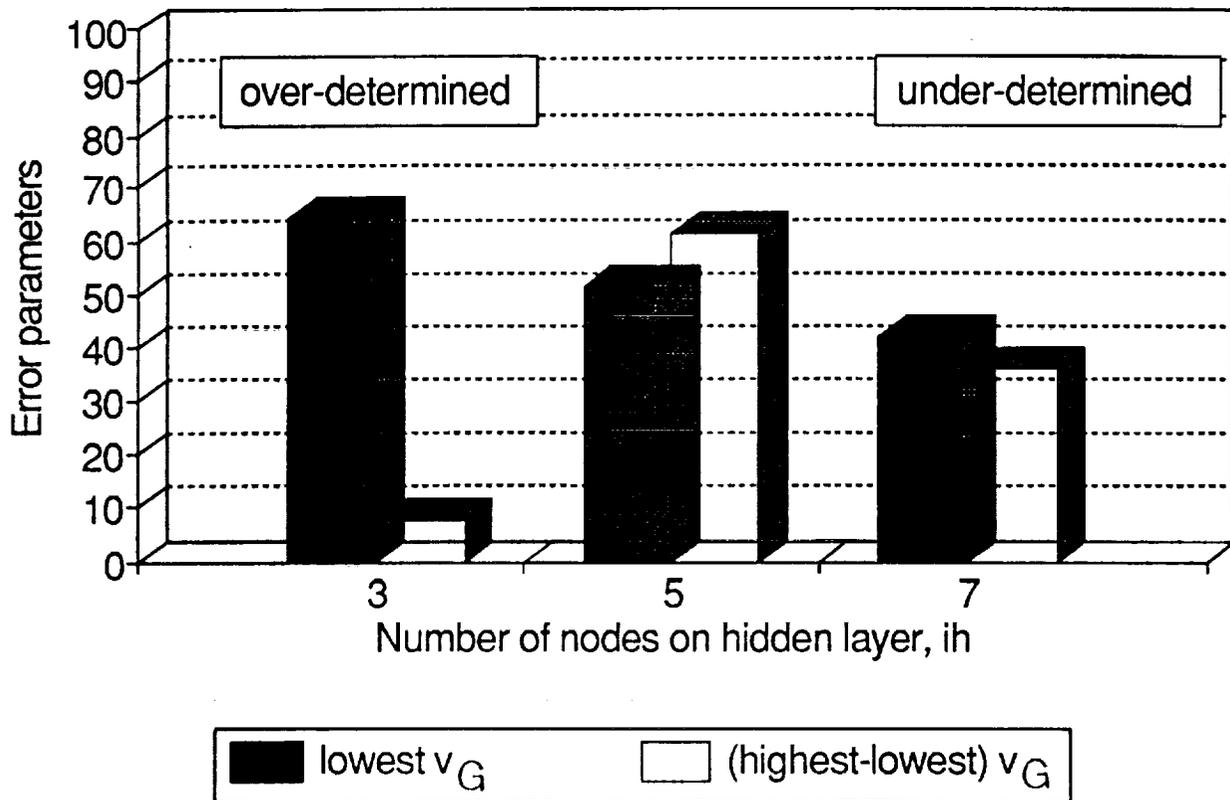
—\*— exact —□— net 1 —×— net 2 —▴— net 3



10. Fox's Banana Function

# Fox's Banana Function

Neural Net Study, 16 training pairs



13. Fox's Banana Function, Error Parameters for 3, 5, and 7 Nodes on the Hidden Layer

# 35 Bar Truss, 4 Design Variables

## Neural Net Study, $ih=3$

